物理数学 I　レポート問題

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1. It is given that $f(z)$ is regular in and on a closed curve C except possibly for a finite numbers of poles within C,and that $|f(z)| \neq 0$ on C.Show from Cauchy's theorem that

$$\frac{1}{2\pi\imath}\oint\frac{f'(z)}{f(z)}dz = N - P$$

where N is the number of zeros and $f(z)$ and P is the number of poles $f(z)$,each counted according to their order,within C.

By tracing the change of $\log f(x)$ around a curve enclosing the first quadrant,show that $f(z) = z^{10} + z^3 + 1$ has three zeros in the first quadrant.

2. Consider the following equation

$$ax^2\frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = 0$$

where a, b and c are constants.This is known as Eular-Cauchy equation.By changing the independent variable from x to t according to $x = e^t$,show that this equation becomes

$$a\frac{d^2y}{dt^2} + (b-a)\frac{dy}{dt} + cy = 0$$

Beginning with two solutions of this equation for each of three cases discussed in the lecture on the second order homogeneous differential equations with constant doefficients,determine the corresponding solutions of Eular-Cauchy equation as a function of x.

3. (*Optional for advanced students:*)Prove the existence and uniqueness theorem for initial value problems of first order linear differential equations.Generalize to the existence and uniqueness theorem for nth-order equations.